Electromagnetic moments of quasi-stable baryons

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We address electromagnetic properties of quasi-stable baryons in the context of chiral extrapolations of lattice QCD results. For particles near their decay threshold we show that the application of a small external magnetic field changes the particle's energy in a non-analytic way. Conventional electromagnetic moments are only well-defined when the background field B satisfies $|eB|/2M_* \ll |M_* - M - m|$ where M_* is the mass of the resonance and M, m the masses of the decay products. An application of this situation is the chiral extrapolation of $\Delta(1232)$-isobar electromagnetic properties. We discuss such an extrapolation of the $\Delta(1232)$-isobar magnetic dipole, electric quadrupole and magnetic octupole moments by a covariant chiral effective field theory.

Keywords: electromagnetic moments, chiral extrapolations, resonances, one-photon approximation

I. INTRODUCTION

Conventionally, the energy shift of a spin 1/2 particle in an external magnetic field $\vec{B}$ is given by:

$$\Delta E = -\vec{\mu}\cdot\vec{B} \ , \tag{1}$$

which defines the particle's static magnetic moment $\vec{\mu}$. However, for resonances near their decay threshold it was shown that the application of a small external magnetic field changes the resonance's energy in a non-analytic way [1]. As a consequence, electromagnetic moments of a resonance with mass M_* and decay product masses M and m are only well defined if the condition

$$|eB|/2M_* \ll |M_* - M - m| \tag{2}$$

is met. In order to use the approximation Eq. (1) for resonances the condition expressed by Eq. (2) has to be fulfilled. We sketch the derivation of this relation in the second section of this report.

Fields in which one encounters the above situation are lattice QCD calculations and field theories. Lattice QCD begin to obtain results for the $\Delta(1232)$-isobar electromagnetic moments [2, 3] where the lightest pion mass is around $m_\pi \simeq 300$ MeV and a chiral extrapolation to the physical point is needed. In doing so, one crosses the threshold value $m_\pi = M_\Delta - M_N$, with $M_\Delta - M_N$ the $\Delta(1232)$-nucleon mass gap, for which the right hand side of Eq. (2) is zero. In the third section we discuss the chiral behavior of $\Delta(1232)$-isobar electromagnetic moments by a manifestly covariant chiral effective field theory [4]. Explicitly studied are the $\Delta(1232)$-isobar magnetic dipole, electric quadrupole and magnetic octupole moments where cusps and singularities reflect the non-applicability of Eq. (2) at the point $m_\pi = M_\Delta - M_N$.

II. ANOMALOUS MAGNETIC MOMENT AND ENERGY SHIFT OF A RESONANCE

We consider a model of two spin 1/2 fields Ψ and ψ of masses M_* and M, respectively, interacting with a scalar field ϕ of mass m by:

$$\mathcal{L}_{\rm int} = g\left(\overline{\Psi}\,\psi\,\phi + \overline{\psi}\,\Psi\,\phi^*\right), \tag{3}$$

where g is a Yukawa coupling constant. The leading order electromagnetic vertex corrections to the anomalous magnetic moment (a.m.m.) of Ψ for charged ϕ and ψ are Feynman-diagrams of the type (D3) and (D4) that are depicted in the next section. For the above Lagrangian the double lines in these graphs denote the Ψ, the single lines the ψ and the dotted lines the ϕ. In the following, we concentrate on graph (D3) a discussion of graph (D4) is analogous.

Coupling the photon minimally to ϕ yields the following a.m.m. κ_* together with the conventional energy shift $\Delta\tilde{E}$:

$$\kappa_* = \frac{2g^2}{(4\pi)^2}\int_0^1 dx\,\frac{-(r+x)\,x(1-x)}{x\mu^2 - x(1-x) + (1-x)r^2} \quad , \quad \Delta\tilde{E} = -\frac{\kappa_*}{2}\tilde{B} \ , \tag{4}$$

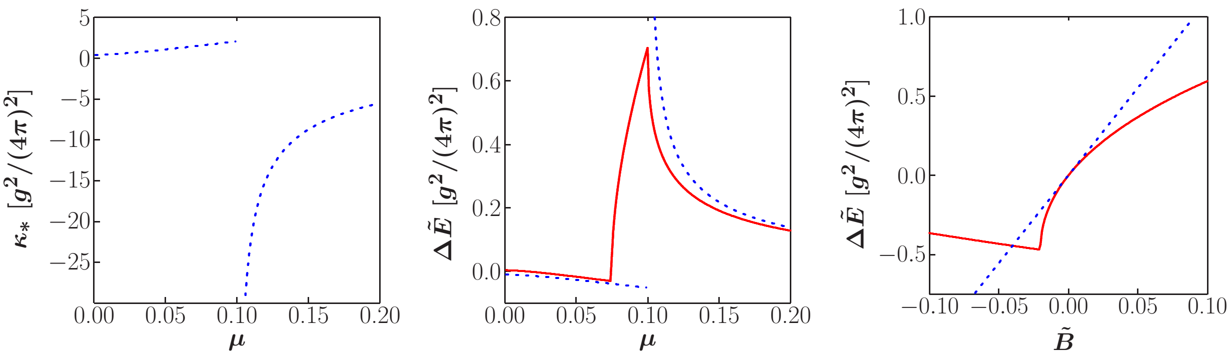

FIG. 1: Left panel: The a.m.m. contribution κ_* as function of $\mu = m/M_*$ for fixed value $r = M/M_* = 0.9$. The dashed blue curve depicts the real part. The imaginary part exhibits a similar singularity that is not shown. Middle panel: Energy shift of the particle Ψ as a function of μ in a constant field of $\tilde{B} = 0.05$ and $r = 0.9$. The dashed blue curve depicts the linear approximation Eq. (4) while the solid red line depicts the solution of Eq. (5). Right panel: Same quantities as in the middle panel this time plotted for constant $\mu = 0.09$ and variable $\tilde{B}$.

with $r = M/M_*$, $\mu = m/M_*$ and the dimensionless variables: $\tilde{B} = \frac{eB_z}{M_*^2}$ and $\Delta\tilde{E} = \frac{\Delta E}{M_*} + \frac{1}{2}\tilde{B}$. In the left panel of Fig. 1 we plot κ_* as a function of the mass parameter μ. We obtain a singularity at the point $m = M_* - M$ ($\mu = 1 - r$), i.e. where m is equal to the mass gap of Ψ and ψ. The same singularity occurs for the graph (D4) with different overall factors. Since an infinite a.m.m. would imply an infinite energy shift of the resonance in an external magnetic field, this singularity can not be physical. The nature of this singularity can be seen by calculating the energy shift of the resonance Ψ with the background field technique of [5].

We consider a constant magnetic field in z-direction, $\vec{B} = B\vec{e}_z$, together with a Ψ spin projection of $s_z = +1/2$. The resulting energy-shift for the case Ψ and ϕ are positively charged is [1]:

$$\Delta\tilde{E} = \frac{g^2}{(4\pi)^2}\Big\{(r+\alpha)(\Omega+\mathcal{A}) - [(r+\alpha)(\Omega+\mathcal{A})]_{\tilde{B}=0}\Big\} \quad , \tag{5}$$

where the $\mathcal{A}$ contribution is analytic in $\tilde{B}$ and the Ω is non-analytic :

$$\mathcal{A} = -2 + \alpha\ln r^2 + \beta\ln\mu^2 - \frac{\mu^2(1-\ln\mu^2) - r^2(1-\ln r^2)}{2(\alpha+r)(1-\tilde{B})} \quad , \quad \Omega = \lambda\,\ln\frac{(\alpha+\lambda)(\beta+\lambda)}{(\alpha-\lambda)(\beta-\lambda)}. \tag{6}$$

with

$$\alpha = \frac{1}{2(1-\tilde{B})}\left(1 + r^2 - \mu^2 - \tilde{B}\right) \quad , \quad \beta = \frac{1}{2(1-\tilde{B})}\left(1 - r^2 + \mu^2 - \tilde{B}\right) \quad , \quad \lambda = \left[\alpha^2 - r^2/(1-\tilde{B})\right]^{1/2} \quad . \tag{7}$$

In the middle panel of Fig. 1 we plot Eq. (4) and Eq. (5). We see that the linear approximation Eq. (4) does not accurately reproduce the energy shift of Ψ in the vicinity of the threshold $m = M_* - M$. Furthermore, plotting Eq. (5) as a function of $\tilde{B}$, right panel of Fig. 1, reveals a cusp that occurs at $\tilde{B} = 0$ for the case $m = M_* - M$. Hence, a definition of the magnetic moment as the derivative at $\tilde{B} = 0$ is not possible. From the λ term in Ω we can see that the energy shift $\Delta\tilde{E}$ can only be expanded, i.e. a magnetic moment defined by Eq. (1), when the condition Eq. (2) is fulfilled. The energy shift $\Delta\tilde{E}$ on the other hand is the physical observable, accessible e.g. in lattice QCD calculations.

III. CHIRAL BEHAVIOR OF $\Delta(1232)$ ELECTROMAGNETIC PROPERTIES

For the $\Delta(1232)$-isobar, a Lorentz-covariant decomposition of the electromagnetic $\gamma\Delta\Delta$ vertex with explicit electromagnetic gauge invariance involves four form factors [6]:

$$\langle\Delta'|V^\mu|\Delta\rangle = -\bar{u}_\alpha(p')\Big\{\left[F_1^*(Q^2)\gamma^\mu + \frac{i\sigma^{\mu\nu}q_\nu}{2M_\Delta}F_2^*(Q^2)\right]g^{\alpha\beta} + \left[F_3^*(Q^2)\gamma^\mu + \frac{i\sigma^{\mu\nu}q_\nu}{2M_\Delta}F_4^*(Q^2)\right]\frac{q^\alpha q^\beta}{4M_\Delta^2}\Big\}u_\beta(p) \quad ,$$

where $u_\alpha(p)$ is the Rarita-Schwinger spinor for a spin-3/2 state with mass M_Δ. At $Q^2 = 0$, these form factors are related to the $\Delta(1232)$-isobar magnetic dipole μ_Δ, electric quadrupole $\mathcal{Q}_\Delta$ and magnetic octupole $\mathcal{O}_\Delta$ moments:

$$\mu_\Delta = \frac{e}{2M_\Delta}\left[e_\Delta + F_2^*(0)\right] \quad , \quad \mathcal{Q}_\Delta = \frac{e}{M_\Delta^2}\left[e_\Delta - \frac{1}{2}F_3^*(0)\right] \quad , \quad \mathcal{O}_\Delta = \frac{e}{2M_\Delta^3}\left[e_\Delta + F_2^*(0) - \frac{1}{2}\left(F_3^*(0) + F_4^*(0)\right)\right] \quad .$$

To study the chiral behavior of these moments, we use the chiral effective Lagrangian given by the BχPT Lagrangian of [7] where the $\Delta(1232)$-isobar is included with the δ-power counting scheme of [8]. The explicit Lagrangian consisting of pion, nucleon, $\Delta(1232)$-isobar and photon fields can be found in [9, 10]. The power counting breaking terms as found in [7] are treated by the renormalization precription of [11], i.e. in addition to the divergent loop contributions also the finite power counting breaking terms are subtracted.

In Fig. 2 we show the Feynman diagrams that contribute to the $\Delta(1232)$-isobar electromagnetic moments at the order p^3 and p^4/Δ with $\Delta = M_\Delta - M_N$.

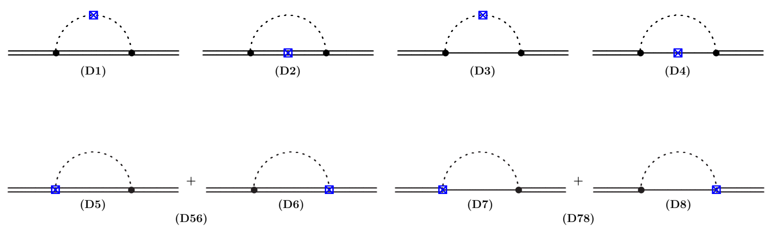

FIG. 2: Diagrams contributing to $\Delta(1232)$ electromagnetic moments. Solid-single lines represent nucleons, solid-double lines the $\Delta(1232)$-isobar and dashed lines the pions. The photon coupling is denoted by blue squares while the $N\pi$ and $\Delta\pi$ vertices by black dots.

In Fig. 3 we show results of our investigation [4]. Depicted are the $\Delta(1232)^+$ magnetic dipole, electric quadrupole and magnetic octupole moments as functions of the pion mass squared m_π^2. The lattice QCD data points are taken from [2] green triangles and [3] orange rectangles. The electromagnetic moments exhibit cusps and singularities at the threshold value $m_\pi = M_\Delta - M_N$, with M_N as the nucleon mass, that stem from the graphs (D3) and (D4). The cusp in the magnetic moment was already seen in Ref. [9]. As discussed in the previous section, this behavior is a consequence of the non-fulfillment of condition Eq. (2) for that m_π value. At this point one can not find any weak magnetic field which enables to define properly the electromagnetic properties of the $\Delta(1232)$-isobar in the conventional way.

For lattice QCD calculations of electromagnetic moments there exist the three point function and the background field methods. In the case of the background field technique, the periodicity condition gives a lower bound for the applied magnetic field strength [2, 12]. Hence, it exists a m_π region around the threshold $m_\pi = M_\Delta - M_N$ in which Eq. (2) is not met. To give explicit situations, we take two spatial lattices of $L = 24$ with spacing $a^{-1} = 2$ GeV and $L = 32$ with spacing $a^{-1} = 1$ GeV and implement the magnetic field by $eBa^2 = 2\pi/L^2$ [2]. Further, we take Eq. (2) to be unity, i.e. a completely non-fulfillment of this relation, and solve for that region. There higher order $\vec{B}$-terms in the energy $E(\vec{B})$ cannot be neglected and a static electromagnetic moment is not well defined in the traditional way. We see that for the given lattices this region ranges from $m_\pi = 275.3$ MeV to $m_\pi = 310.7$ MeV for the coarse lattice and from $m_\pi = 290.5 \sim 295.5$ MeV for the finer lattice. We represent these two regions as grey bands in Fig. 3.

A similar problem could also affect lattice calculations of resonance a.m.m. by the three point function method. Here the results are limited by finite Q^2 and an extrapolation to $Q^2 = 0$ needs to be done. Qualitatively, a finite Q^2 would enter as an additional energy paramter and the singularities would be shifted away from the point $Q^2 = 0$ for pion masses other than $M_\Delta - M_N$. Hence, for a given m_π one would get a finite Q^2 value for which the form factor is singular. For practical lattice calculations this could mean that one extrapolates across this singularity to $Q^2 = 0$ when all data points are on the right of the singularity.

IV. CONCLUSION

We addressed electromagnetic properties of quasi-stable particles. For such particles we showed that the application of a small external magnetic field changes the particle's energy in a non-analytic way. Static electromagnetic moments are only well defined when the condition Eq. (2) is fulfilled [1]. Explicit situations where Eq. (2) could be violated are lattice QCD calculations of electromagnetic properties and their chiral extrapolations by means of effective-field theories. We discussed such an extrapolation for the $\Delta(1232)$-isobar electromagnetic moments by means of a

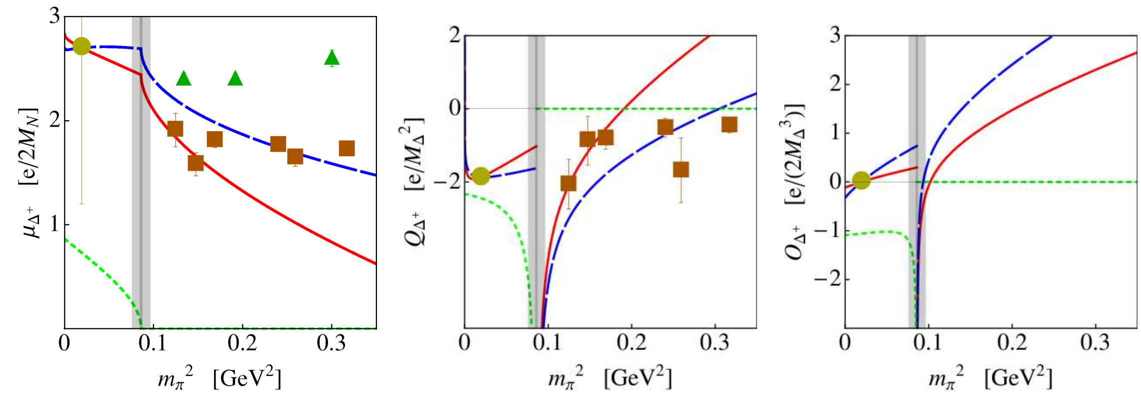

FIG. 3: The magnetic dipole μ_{Δ^+}, electric quadrupole $\mathcal{Q}_{\Delta^+}$ and magnetic octupole $\mathcal{O}_{\Delta^+}$ moments of the $\Delta^+(1232)$. The curves correspond to: red solid and blue long-dashed curves show our results with and without inclusion of $\gamma\Delta\Delta$ non-minimal couplings, respectively, and are constraint to values at the physical pion mass; green short-dashed curve to the imaginary parts. The yellow circle corresponds to the used values at the physical point: the experimental value $\mu_{\Delta^+} = (2.7 \pm 1.5)\ \mu_N$, a large-$N_c$ estimate $\mathcal{Q}_{\Delta^+} = -1.87\ e/M_\Delta^2$ and $\mathcal{O}_{\Delta^+} = 0$. The lQCD data of [2] are denoted by green triangles while those of [3] are depicted by orange rectangles. The grey bands are described in the text.

covariant chiral effective field theory [4]. The non-fulfillment of Eq. (2) is reflected by cusps and singularities at the point $m_\pi = M_\Delta - M_N$. Modern lattice QCD results are about to approach the pion mass region where this condition applies and the presented techniques can be used.

Acknowledgments

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